4. Find $x$ and $y$ so that the quadrilateral is a parallelogram.

\[ 4x - 17 = 2x - 1 \]
\[ 3y + 5 = 3y - 6 \]

\[ x = 8 \]
\[ y = 9 \]

Draw a labeled diagram. Show all work.

5. In parallelogram $ABCD$, $\angle A = (2x + 50)^\circ$ and $\angle C = (3x + 40)^\circ$. Find $\angle B$.

\[ 2x + 50 = 3x + 40 \]
\[ x = 10 \]
\[ \angle A = 2(10) + 50 = 70 \]
\[ \angle B = 180 - 70 = 110^\circ \]

6. In parallelogram $ABCD$, $\angle A = (2x - 10)^\circ$ and $\angle B = (5x + 15)^\circ$. Find $\angle C$.

\[ 5x + 15 + 2x - 10 = 180 \]
\[ x = 25 \]
\[ \angle A = 2(25) - 10 = 40^\circ \]
\[ \angle C = 40^\circ \]

7. In parallelogram $ABCD$, diagonals $\overline{AC}$ and $\overline{BD}$ intersect at $E$. If $BE = 4x - 12$ and $DE = 2x + 8$, find $x$ and $BD$.

\[ 4x - 12 = 2x + 8 \]
\[ x = 10 \]
\[ BD = \sqrt{(4(10) - 12)^2} \]
\[ = 28(2) \]
\[ BD = 56 \]

8a) Find the value of $x$ and $y$ that will make $ABCD$ a parallelogram.

\[ m\angle B = (2y - 3x)^\circ \]
\[ m\angle C = (x + y)^\circ \]
\[ m\angle D = (5x - y)^\circ \]

\[ x + y + 5x - y = 180 \]
\[ x = 30 \]
\[ \angle B = 2(30) - 3(30) = 70 \]
\[ \angle D = 5(30) - 80 = 70 \]
\[ \angle C = 30 + 80 = 110 \]
\[ \angle A = 110^\circ \]

b) Then find the measure of each angle of the parallelogram.

\[ m\angle B = (2y - 3x)^\circ \]
\[ m\angle C = (x + y)^\circ \]
\[ m\angle D = (5x - y)^\circ \]
28

Name ____________

Date ____________

CC Geometry H

HW#28

1) In quadrilateral ABCD, AB = 8, BC = 6, CD = 8, DA = 6, AC = 10, and BD = 10.
   Draw a labeled diagram
   a) State why ABCD is a parallelogram.  b) State why □ABCD is a rectangle.
      Both pairs of opp. sides are equal.

A □ with equal diag. rectangle.

2) Based on the markings in each diagram, name the quadrilateral as specifically as possible.

Sample:

Answer: Square
(Equilateral quad. is a rectangle.
Rectangle with 2 = adj. sides is a square)

a) rhombus  b) square  c)  d) rhombus

3) Given: \( \overline{EF} \), \( \angle 2 \) is supplementary to \( \angle 1 \), \( \angle C \equiv \angle 1 \)
   Prove: \( \square ABCD \) is a parallelogram

<table>
<thead>
<tr>
<th>statement</th>
<th>reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overline{EF} ), ( \angle 2 ) is suppl. to ( \angle 1 ), ( \overline{BC} \parallel \overline{AB} )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \overline{DC} \parallel \overline{BA} )</td>
<td>2. When 2 lines are cut by a transversal, such that alt. int. ( \angle s ) are ( \equiv ), the lines are ( \parallel )</td>
</tr>
<tr>
<td>3. ( \angle 1 ) is suppl. to ( \angle 3 )</td>
<td>3. Lin. pairs form suppl. ( \angle s )</td>
</tr>
<tr>
<td>4. ( \angle 2 \equiv \angle 3 )</td>
<td>4. Supplements of the same ( \angle ) are ( \equiv )</td>
</tr>
<tr>
<td>5. ( \overline{AD} \parallel \overline{CB} )</td>
<td>5. When 2 lines are cut by a transversal, such that corr. ( \angle s ) are ( \equiv ), the lines are ( \parallel )</td>
</tr>
<tr>
<td>6. ( \square ABCD ) is a □</td>
<td>6. A quad. w/ both pairs of opp. sides ( \parallel )</td>
</tr>
</tbody>
</table>
4) Which criteria for triangle congruence cannot be used to prove the two shaded triangles pictured in this parallelogram are congruent?

(1) ASA  (2) SSS  (3) SAS  (4) HL

5) Given: Isosceles \( \triangle ABC \), \( \overline{CD} \) bisects vertex \( \angle C \), \( \overline{CD} = \overline{DE} \)

Prove: a) \( \triangle ACD \cong \triangle BCD \)
   b) \( \text{ACBE is a parallelogram} \)

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<tr>
<td>1. Isos. ( \triangle ABC ), ( \overline{CD} ) bisects vertex ( \angle C ), ( \overline{CD} = \overline{DE} )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle 1 \cong \angle 2 )</td>
<td>2. An ( \angle ) bis. divides an ( \angle ) into 2 ( \angle ) halves.</td>
</tr>
<tr>
<td>3. ( \overline{CD} \cong \overline{CD} )</td>
<td>3. Reflexive Prop.</td>
</tr>
<tr>
<td>4. ( \overline{CA} \cong \overline{CB} )</td>
<td>4. An Isos. ( \triangle ) has 2 ( \angle ) sides.</td>
</tr>
<tr>
<td>5. ( \triangle ACD \cong \triangle BCD )</td>
<td>5. SAS</td>
</tr>
<tr>
<td>6. ( \overline{AD} \cong \overline{BD} )</td>
<td>6. Corr. parts of ( \cong ) ( \triangle )s are ( \cong ).</td>
</tr>
<tr>
<td>7. ( \overline{CE} ) and ( \overline{BF} ) bis. each other.</td>
<td>7. Seg. bisectors divide seg. into 2 ( \cong ) halves.</td>
</tr>
<tr>
<td>8. ( \text{ACBE is a} \bigcirc )</td>
<td>8. A quad. whose diag. bisect each other.</td>
</tr>
</tbody>
</table>

6) Square \( ABCD \) with diagonals \( \overline{AC} \) and \( \overline{BD} \) intersecting at \( E \) is drawn. \( AC = 14x - 34 \) and \( BE = 5x - 7 \).

a) Find \( BD \).  
   b) Find \( CD \), in simplest radical form.

\[
2(5x-7) = 14x-34 \\
10x-14 = 14x-34 \\
20 = 4x \\
5 = x \\
\text{AC} = \text{BD} = 14(5) - 34 = \boxed{36 \text{ units}}. \\
18^2 + 18^2 = DC^2 \Rightarrow \sqrt{648} = DC \Rightarrow 18\sqrt{2} = DC.
\]
Aim #29: How do we prove a parallelogram is a rectangle?

Do Now: 1) A rectangle is a parallelogram: Always/Sometimes/Never

2) Which is not true about a rectangle?
   a) Diagonals bisect each other.
   b) Opposite angles are congruent.
   c) Diagonals bisect the angles.
   d) Diagonals are congruent.

3) In rectangle ABCD, AE = 3x + y, EC = 2x + y + 7 and DE = 2y + 3x - 1. Find the values of x and y.

   \[ 3x + y = 2x + y + 7 \]
   \[ 2y + 3x - 1 = 3x + y \]

   \[ x = 7 \]

   \[ 2y - 1 = y \]
   \[ -1 = -y \]
   \[ y = 1 \]

Proving a property of a rectangle:

If a parallelogram is a rectangle, then its diagonals are congruent.

Given: Rectangle GHIJ
Prove: GI \cong HJ

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<tr>
<td>1. Rect. GHIJ</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. \angle GJ \cong \angle HIJ</td>
<td>2. A rect. is equiangular.</td>
</tr>
<tr>
<td>3. \overline{JI} \cong \overline{IJ}</td>
<td>3. Reflexive Prop.</td>
</tr>
<tr>
<td>4. \overline{GJ} \cong \overline{HI}</td>
<td>4. Opp. sides of a rect. are \cong.</td>
</tr>
<tr>
<td>5. \triangle GJ \cong \triangle HIJ</td>
<td>5. SAS</td>
</tr>
<tr>
<td>6. \overline{GI} \cong \overline{HI}</td>
<td>6. Corr. parts of \cong \triangle s are \cong.</td>
</tr>
</tbody>
</table>
Given: Rect. RSTU, M is midpoint of RS
Prove: ΔUMT is isosceles

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<td>1. Rect. RSTU, M is midpt of RS</td>
<td>0. Given</td>
</tr>
<tr>
<td>2. RM = SM</td>
<td>2. A midpt divides a seg. into 2 ± halves.</td>
</tr>
<tr>
<td>3. RU = ST</td>
<td>3. Opp. sides of a rect. are ±.</td>
</tr>
<tr>
<td>4. XR = XS</td>
<td>4. A Rect. is equiangular.</td>
</tr>
<tr>
<td>5. ΔRUM ≅ ΔSTM</td>
<td>5. SAS</td>
</tr>
<tr>
<td>7. ΔUMT is isos.</td>
<td>7. A Δ w/ 2 ± sides.</td>
</tr>
</tbody>
</table>

1a) In Quadrilateral ABCD, AE = 7x - 1, and EC = 5x + 5. Find AC.

\[
7x - 1 = 5x + 5 \\
2x = 6 \\
x = 3 \\
\[
AC = 7(3) - 1 + 5(3) + 5 = 40u.
\]

b) If DB = 10x + 10, find DB.

\[
10(3) + 10 = 40u. 
\]

c) What kind of parallelogram is ABCD and justify your response.

rect. → a Rect. w/ 2 ± diagonals.

2) The length of a rectangle is seven more than the width. A diagonal is one more than twice the width. Find the width, length and the length of the diagonal using an algebraic solution.

\[
a^2 + b^2 = c^2 \\
(w + 7)^2 + (w)^2 = (2w + 1)^2 \\
(w + 7)(w + 7) + w^2 = (2w + 1)(2w + 1) \\
w^2 + 14w + 49 + w^2 = 4w^2 + 4w + 1 \\
dw^2 + 14w + 49 = 4w^2 + 4w + 1 \\
D = 2w^3 - 10w - 48 \\
D = w^2 - 5w - 24 \\
D = (w - 3)(w^2 + 1) \\
w = 8 \\
\frac{w}{w - 3}
To prove a parallelogram is a rectangle, prove one of the following:

1. it has 1 rt. \( \angle \).

2. diagonals are \( \perp \).

3. ***An equiangular quadrilateral is a rectangle.***

---

**Given:** \( \square ABCD, CE \perp EA, BF \perp EA \)

**Prove:** ECBF is a rectangle

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<td>1. ( \square ABCD, CE \perp EA, BF \perp EA )</td>
<td>1. <strong>Given</strong></td>
</tr>
<tr>
<td>2. ( AD \parallel BC ) ( \text{or } FE \parallel BC )</td>
<td>2. Opp. sides of a ( \square ) are ( \parallel ).</td>
</tr>
<tr>
<td>3. ( CE \parallel BF )</td>
<td>3. 2 seg. ( \perp ) to the same seg. are ( \parallel ) to each other.</td>
</tr>
<tr>
<td>4. ECBF is a ( \square ). ( \checkmark )</td>
<td>4. A ( \square ) w/ both pairs of opp. sides ( \parallel ).</td>
</tr>
<tr>
<td>5. ( \angle XE ) is a rt. ( \angle ). ( \checkmark )</td>
<td>5. ( \perp ) lines form rt ( \angle )s.</td>
</tr>
<tr>
<td>6. ECBF is a rect.</td>
<td>6. A ( \square ) w/ one rt. ( \angle ).</td>
</tr>
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---
Given: Rect. ABCD
Prove: $\angle CAD \equiv \angle BDA$

---

Given: Rect. PQRS
Prove: $\angle 1 \equiv \angle 2$
1) Rectangle $ABCD$ is shown below. Find $x$:

2) The length of two adjacent sides of a rectangle differ by 17. If the perimeter of the rectangle is 146, compute a diagonal and the area of the rectangle. Solve algebraically.

3) Given: Rect. $ABCD$, $\overline{AP} \cong \overline{DN}$
   Prove: a) $\triangle ABP \cong \triangle DCN$
   b) $\overline{AE} \cong \overline{DE}$

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<tr>
<td>$\angle BAP \cong \angle DCP$</td>
</tr>
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<td>$\angle ABP \cong \angle DCN$</td>
</tr>
<tr>
<td>$\overline{AE} \cong \overline{DE}$</td>
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**CC Geometry H**
**HW #29**
Mixed review:
1) Construct the following using a compass and straightedge:
   a. Median from vertex $A$
   b. Altitude from vertex $A$. 

![Diagram of median and altitude construction]

4) In rectangle $ABCD$ shown below, $AC$ and $BD$ are diagonals. If $m\angle 1 = 49$, find $m\angle ADB$.

![Diagram of rectangle with diagonals]

For #s 5 and 6, refer to rectangle $ABCD$ shown below, with diagonals $AC$ and $BD$ intersecting at $R$.

5) If $DR = 4(3x - 10)$ and $CR = 3(x - 2) + 12$, find $x$, $AR$, $AC$, and $BD$.

![Diagram with point $R$]

6) If $AC = 3(2x + 5) - \frac{1}{4}(4x + 4)$ and $BD = \frac{2}{3}(12x - 3) + 5x$, find $x$, $AC$, and $DR$.

![Diagram with expressions for $AC$ and $BD$]