Aim #82: What is compound interest?
Homework: Handout

Do Now: Kyra invested $15,000 of her money at a 1.8% per year bank rate. Write an equation that represents Kyra’s account balance after $t$ years. What is Kyra’s account balance, to the nearest cent, after:

- Year 1?

- Year 2?

- Year 3?

- Year 4?

- Year 5?

- Compounded interest adds interest not only to your principle amount, but your accumulated interest over time.

- To calculate compound interest we use the formula below where $A = \text{total balance } \text{after } t \text{ years}, P = \text{principal amount (amount borrowed or invested)}, r = \text{interest rate (decimal form), and } t = \text{time in years}.$

<table>
<thead>
<tr>
<th>Year</th>
<th>Equation</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$15,270$</td>
<td>$15,270$</td>
</tr>
<tr>
<td>2</td>
<td>$15,544.86$</td>
<td>$15,544.86$</td>
</tr>
<tr>
<td>3</td>
<td>$15,824.67$</td>
<td>$15,824.67$</td>
</tr>
<tr>
<td>4</td>
<td>$16,109.50$</td>
<td>$16,109.50$</td>
</tr>
<tr>
<td>5</td>
<td>$16,399.47$</td>
<td>$16,399.47$</td>
</tr>
</tbody>
</table>

1) Jack has $500 to invest. The bank offers an interest rate of 6% compounded annually.

a) How much money will Jack have after 1 year?

$$A = 500 \left(1 + .06\right)^1 = \$530$$

b) How much money will Jack have after 5 years?

$$A = 500\left(1+.06\right)^5 = \$669.11$$

c) How much money will Jack have after 10 years?

$$A = 500\left(1+.06\right)^{10} = \$895.42$$
2) Suppose you go to the bank and invest $6,000. Find the account balance to the nearest cent after 15 years paying 4%:

a) compounded annually

\[ A = 6000 \left(1 + \frac{0.04}{1}\right)^{15 \cdot 1} = \$10,805.66 \]

b) compounded semi-annually

\[ A = 6000 \left(1 + \frac{0.04}{2}\right)^{15 \cdot 2} = \$10,868.17 \]

c) compounded quarterly

\[ A = 6000 \left(1 + \frac{0.04}{4}\right)^{15 \cdot 4} = \$10,900.18 \]

d) Based on your previous answers for parts a through c, would you rather invest your money annually, semi-annually or quarterly?

Quarterly

To calculate the total amount of money compounded monthly or quarterly we use the formula below where \( n \) = number of times interest is compounded per year.

\[ A = P \left(1 + \frac{r}{n}\right)^{nt} \]

3) You have $200 to invest for 10 years. Is it monetarily better to invest your money in a bank that pays 4.05% interest compounded annually or 4% interest compounded quarterly?

\[ A = 200 \left(1 + \frac{0.0405}{1}\right)^{10} \]

= $297.48

\[ A = 200 \left(1 + \frac{0.04}{4}\right)^{10 \cdot 4} \]

= $297.77

towards

better

investment
4) A youth group has a yard sale to raise money for a charity. The group earns $800 but decided to put its money in the bank for a while. Calculate the amount of money to the nearest cent the group will have if:

a) Hot Bank pays compound quarterly interest at a rate of 3% and the youth group leaves the money in for 5 years.

\[ A = 800 \left(1 + \frac{.03}{4}\right)^{5 \times 4} = \$928.95 \]

b) Cool Bank pays compound monthly interest at a rate of 2% and the youth group leaves the money in for 7 years.

\[ A = 800 \left(1 + \frac{.02}{12}\right)^{7 \times 12} = \$920.11 \]

5) Suppose you deposit $3,000 into an account that earns 6% compounded annually interest. Find the balance of the account to the nearest cent after 24 months:

\[ y = 3000(1 + .06)^2 = \$3370.80 \]

6) Suppose your parents deposited $1500 in an account paying 3.6% interest when you were born. Find the account balance to the nearest cent after 18 years:

a) compounded semi-annually

\[ A = 1500 \left(1 + \frac{.036}{2}\right)^{18 \times 2} = \$2851.09 \]

b) compounded monthly

\[ A = 1500 \left(1 + \frac{.036}{12}\right)^{18 \times 12} = \$2864.79 \]

c) compounded every 4 months

\[ A = 1500 \left(1 + \frac{.036}{3}\right)^{18 \times 3} = \$2856.53 \]
7) Joe wants the money he deposits in an account to amount to $3000 at the end of three years. If the annual interest rate is 4.2%, how much money, to the nearest cent, should he deposit when:
a) interest is compounded annually
\[ 3000 = P \left(1 + \frac{.042}{1}\right)^3 \]
\[ 3000 = \frac{P \left(1 + .042\right)^3}{1} \]
\[ \frac{3000}{1.1265166} = P \]
\[ \approx 2651.66 \]
b) interest is compounded semi-annually
\[ 3000 = P \left(1 + \frac{.042}{2}\right)^6 \]
\[ \frac{3000}{2.64830} = P \]
c) interest is compounded monthly
\[ 3000 = P \left(1 + \frac{.042}{12}\right)^{36} \]
\[ \frac{3000}{21645.43} = P \]

8) Suppose you needed $21,000 in 5 years. You want to invest at 2.4% interest, compounded monthly. How much should you invest?
\[ 21000 = P \left(1 + \frac{.024}{12}\right)^{60} \]
\[ \frac{21000}{18,627.56} = P \]
\[ \approx 1110.40 \]

9) Suppose that you plan to need $10,000 in thirty-six months; time when your child starts attending university. You want to invest at a bank that offers 3.5% interest, compounded semi-annually. How much should you invest?
\[ 10000 = P \left(1 + \frac{.035}{2}\right)^{36} \]
\[ \frac{10000}{9011.43} = P \]
\[ \approx 1.11 \]

10) Craig invested $600 in a savings account at a 1\(\frac{3}{2}\)% annual interest rate. He made no deposits or withdrawals on the account for 2 years. The interest was compounded annually. Find, to the nearest cent, the balance in the account after two years.
\[ A = 600 \left(1 + .0175\right)^2 \]
\[ = \$621.18 \]

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Sum it Up!

Compound Interest - Interest is calculated once per period on the current amount borrowed or invested. Each period, the interest becomes a part of the principal.