Aim #45: Rectilinear motion (revisited)

Do Now: Given \( s(t) = 3t^3 - 6t^2 + 3t + 1 \), find

\[
\begin{align*}
    v(t) &= s'(t) = 9t^2 - 12t + 3 \\
    a(t) &= s''(t) = v'(t) = 18t - 12
\end{align*}
\]

eg. At what time does the particle change direction?

\[
    v(t) = 9t^2 - 12t + 3 = 3(3t^2 - 4t + 1)
    = 3(3t - 1)(t - 1) = 0
\]

eg. When is the particle moving left?

\[
    \left( \frac{1}{3}, 1 \right)
\]

eg. Moving right?

\[
    (-\infty, \frac{1}{3}) \cup (1, \infty)
\]

\[
    v(t) \quad + \quad - \quad + \quad + \quad + \quad + \quad +
    \frac{1}{3} \quad 1
\]
eg. What is the particle's displacement from time $t = 0$ to $t = 3$?

- $s(0) = 1$
- $s(3) = 81 - 54 + 9 + 1 = 37$

eg. What is the total distance traveled by the particle from time $t = 0$ to $t = 3$?

- $s(0) = 1$
- $s(\frac{1}{3}) = \frac{1}{9} - \frac{6}{9} + 1 + 1 = 1 \frac{4}{9}$
- $s(1) = 1$
- $s(3) = 37$

**Answer**

- $s(3) = 37$
- $s(\frac{1}{3}) = 1 \frac{4}{9}$
- $s(1) = 1$
- $s(0) = 1$

What if we didn’t know the position function?

eg. $v(t) = 9t^2 - 12t + 3$

Integration is an area accumulator

\[
\int_0^3 (9t^2 - 12t + 3) \, dt = \left[ \frac{9t^3}{3} - \frac{12t^2}{2} + 3t \right]_0^3 = \frac{81 - 54 + 9}{3} - (0) = 36
\]

\[
\int_0^3 v(t) \, dt = 36
\]

**Hint**

In this example, it is total displacement
\[
\int_{0}^{1/3} v(t) \, dt = \left. \frac{4}{9} \right|_{0}^{1/3} \int_{1/3}^{1} v(t) \, dt = \left. \frac{4}{9} \right|_{1/3}^{1} \int_{1}^{3} v(t) \, dt = 36
\]

Total distance traveled: \(36 \times \frac{4}{9}\)

Place \(v(t)\) in \(y_1\)

Math #9

Use ALPHA F4 to get \(y_1\)

\((y_1, x, \#)\#\)

**Example.** A particle moves along the x-axis with position at time \(t\) given by \(x(t) = e^{-t}\sin t\) for \(0 \leq t \leq 2\pi\).

Find the time \(t\) at which the particle is farthest to the left. Justify your answer.

\[v(t) = -e^{-t}\cos t + \sin t \cdot e^{-t} \cdot 1\]

\(t = 0.785\)

\(= 3.927\)

\(v(t)\) changes sign at \(0.785\) and \(3.927\), indicating the particle changes direction. The particle is farthest to the left at \(3.927\) with \(x(3.927) = -0.014\).
A squirrel starts at building A and time $t = 0$ and travels along a straight, horizontal wire connected to building B. For $0 \leq t \leq 18$, the squirrel’s velocity is modeled by the piecewise-linear function defined by the graph above.

(a) At what times in the interval $0 \leq t \leq 18$, if any, does the squirrel change direction? Give a reason for your answer.

$v(t)$

$t = 9$ and $t = 15$

because $v(t)$ changes from $+$ to $-$ and from $-$ to $+$
(b) At what time in the interval 0 ≤ t ≤ 18 is the squirrel farthest from building A? How far from building A is the squirrel at that time?

\[ t = 9 \quad x(9) = 140 \]
\[ x(5) = 90 \]
\[ x(18) = 115 \]

(c) Find the total distance the squirrel travels during the time interval 0 ≤ t ≤ 18.

\[ 140 + 50 + 25 = 215 \]

(d) Write expressions for the squirrel’s acceleration \( a(t) \), velocity \( v(t) \), and distance \( x(t) \) from building A that are valid for the time interval 7 < t < 10.

\[ a(t) = -10 \]
\[ v(t) = -10t + 90 \]
\[ y - 20 = -10(x - 7) \]
\[ y = -10x + 90 \]
\[ \int_{7}^{t} v(t) \, dt = \left[ -10t + 90 \right]_{7}^{t} + 120 \]
\[ x(t) = \left[ -5t^2 + 90t \right]_{7}^{10} + 120 \]
\[ = -5t^2 + 90t - 265 \]
A particle moves along the x-axis so that its velocity at time $t$, for $0 \leq t \leq 6$, is given by a differentiable function $v$ whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t-axis and the graph of $v$ on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.

(a) For $0 \leq t \leq 6$, find both the time and position of the particle when the particle is farthest to the left. Justify your answer.

$x(0) = -2$
$x(3) = -10$
$x(5) = -7$
$x(6) = -9$

$t = 3$
(b) For how many values of \( t \), where \( 0 \leq t \leq 6 \), is the particle at \( x = -8 \)? Explain your reasoning.

\[
\begin{align*}
x(0) &= -2 \\
x(3) &= -10 \\
x(5) &= -7 \\
x(6) &= -9
\end{align*}
\]

3 times IVT

(c) On the interval \( 2 < t < 3 \), is the speed of the particle increasing or decreasing? Give a reason for your answer.

\( v(t) < 0 \) \hspace{1cm} \text{slowing down}

\( a(t) > 0 \)
(d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

\[ a(t) \]

\[ \begin{array}{ccccccc}
0 & + & + & + & + & - & - & - \\
1 & 2 & 3 & 4 & 5 & 6 \\
\end{array} \]

\[ (0, 1) \cup (4, 6) \]

Slope of velocity is negative.

(b) Given:

\[ \frac{dV}{dt} = -\frac{\pi h}{3} \]

\[ V = \frac{1}{3} \pi r^2 h \]

a) \[ V = \frac{1}{3} \pi \left( \frac{h}{3} \right)^2 h \]

\[ V = \frac{1}{27} \pi h^3 \]

b) \[ \frac{dV}{dt} = \frac{4}{3} \pi h^2 \cdot \frac{dh}{dt} \]

\[ -\frac{\pi h}{3} = 4\pi h^2 \cdot \frac{dh}{dt} \]

\[ -\frac{3}{h} = \frac{dh}{dt} \]

\[ \frac{h}{r} = \ \frac{12}{4} \]

\[ 12r = 4h \]

\[ r = \frac{h}{3} \]

\[ 729 = h^3 \]

\[ h = 9 \]

\[ \frac{dh}{dt} = -\frac{3}{9} = -\frac{1}{3} \]

\[ \text{W/m}^2 \]
Caren rides her bicycle along a straight road from home to school, starting at home at time $t = 0$ minutes and arriving at school at time $t = 12$ minutes. During the time interval $0 \leq t \leq 12$ minutes, her velocity $v(t)$, in miles per minute, is modeled by the piecewise-linear function whose graph is shown above.

(a) Find the acceleration of Caren’s bicycle at time $t = 7.5$ minutes. Indicate units of measurement.

$$a(t) = -1 \text{ miles}/\text{min}^2$$

(b) Using correct units, explain the meaning of $\int_0^{12} |v(t)| \, dt$ in terms of Caren’s trip. Find the value of $\int_0^{12} |v(t)| \, dt$.

Total distance travelled by Caren from $t = 0$ to $t = 12$ minutes:

$$0.2 + 2 + 1.5 + 3 + 2.5 + 6 + 1 = 1.8 \text{ miles}$$
(c) Shortly after leaving home, Caren realizes she left her calculus homework at home, and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer.

\[ t = 2 \quad \text{because} \quad v(t) \text{ goes from positive to negative.} \]

(d) Larry also rides his bicycle along a straight road from home to school in 12 minutes. His velocity is modeled by the function \( w(t) = \frac{\pi}{15} \sin \left( \frac{\pi}{12} t \right) \), where \( w(t) \) is in miles per minute for \( 0 \leq t \leq 12 \) minutes. Who lives closer to school: Caren or Larry? Show the work that leads to your answer.

Caren: \[ \int_0^{12} v(t) \, dt = 1.4 \text{ miles} \]

Larry: \[ \frac{\pi}{15} \int_0^{12} \sin \left( \frac{\pi}{12} t \right) \, dt \]

\[ = \frac{4}{5} \left( -\cos \left( \frac{\pi}{12} t \right) \right) \bigg|_0^{12} \]

\[ = \frac{4}{5} \left( \cos 0 - (-\cos 0) \right) \]

\[ = \frac{4}{5} \left( 1 + 1 \right) = \frac{8}{5} = 1.6 \text{ miles} \]

Homework: Homework #45 - sheet